Part C Major Option Astrophysics

High-Energy Astrophysics

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Today's lecture: Accretion

- Revision of the Eddington luminosity and accretion rate.
- Accretion discs.
- Properties of the thin accretion disc.
- Evidence for accretion onto black holes (will probably over-run to the next lecture).

The Eddington Luminosity

Today we will consider the accepted mechanism for the production of the extreme luminosities of active galaxies: accretion of material onto supermassive black holes. First we will consider the *Eddington accretion limit.*

The Eddington *luminosity* was introduced in B3 astrophysics in the context of massive stars. The notion is very simple: for *any* object in the depths of space, there is a maximum luminosity beyond which radiation pressure will overcome gravity, and material outside the object will be forced away from it rather than falling inwards.

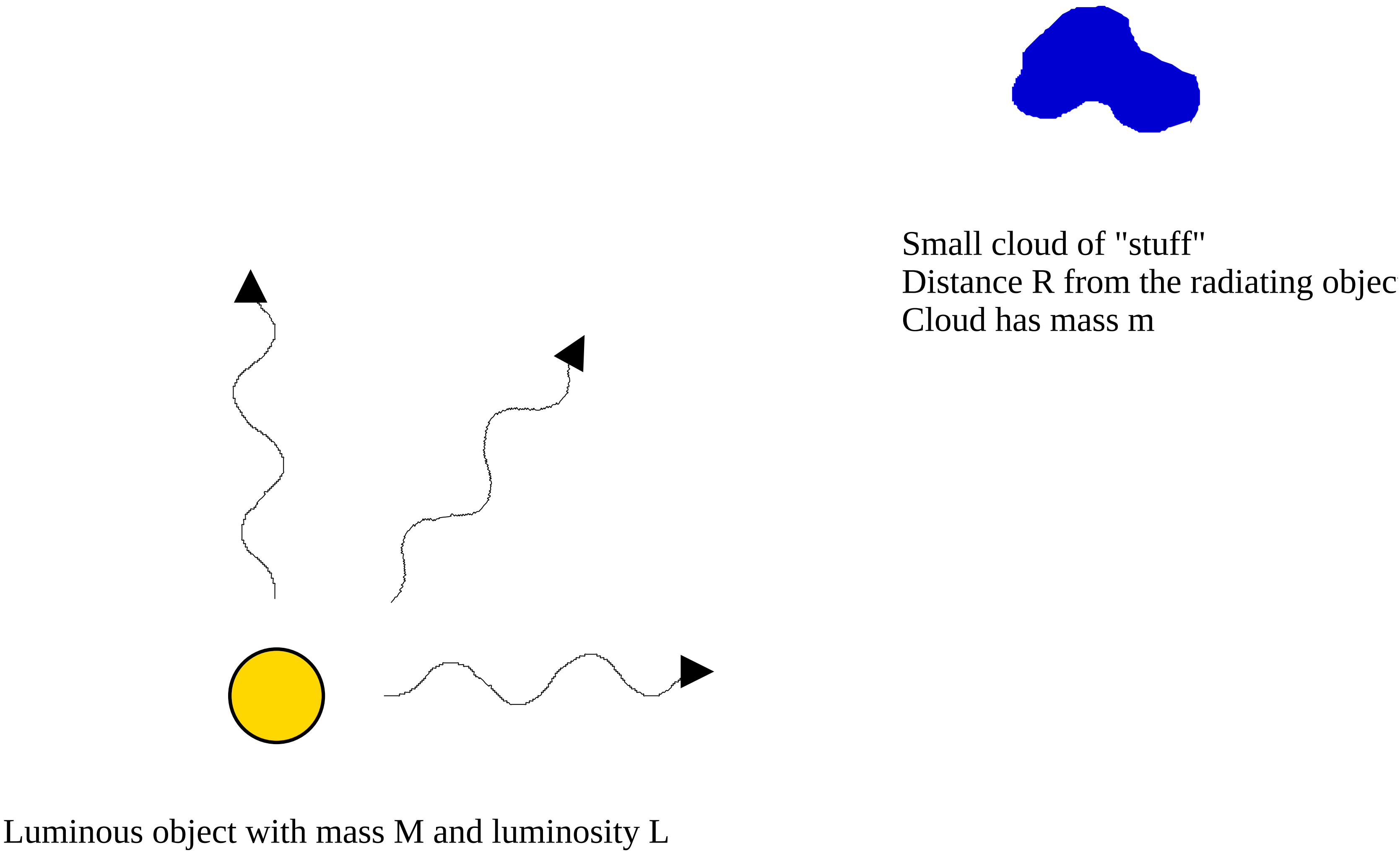
Sir Arthur Stanley Eddington (1882-1944)

Journalist: “Sir Arthur, it is said that only three people in the world understand relativity!”

Eddington: “Yes I’ve heard that. I am trying to work out who the third person is...”

The ingredients we need to derive the Eddington luminosity are:

- The mass of the central object, M
- The total luminosity, L
- A suitable *opacity* for radiation pressure against any surrounding material. We will see that this must have the dimensions of *area per unit mass*.


Small cloud of "stuff"
Distance R from the radiating objec
Cloud has mass m
Luminous object with mass M and luminosity L

We wish to find the luminosity at which the gravitational force inwards balances with the radiation force outwards. The gravitational force is given simply via:

$$F_{\text{grav}} = \frac{GMm}{R^2}$$

To calculate the radiation force first we need to get the *radiation pressure* at R:

$$P_{\text{rad}} = \frac{L}{c}\frac{1}{4\pi R^2}$$

Then to calculate the radiation force on the cloud, we need its opacity, κ. Radiation pressure is force per unit area; opacity is the cross-sectional area per unit mass for radiation scattering.

$$F_{\text{rad}} = P_{\text{rad}}\kappa m$$

Balancing the two forces gives:

$$\frac{GMm}{R^2} = \frac{L\kappa m}{4\pi R^2 c}$$

And solving for this luminosity we get:

$$L = \frac{4\pi GMc}{\kappa}$$

Some important things to note here are:

- The Eddington luminosity depends only on the mass of the radiating object.

- We have assumed spherical symmetry.

Now, we have yet not specified a particular value for κ. In high-energy accretion scenarios we make a useful approximation on the basis that the accreting material is mostly ionized hydrogen and the opacity is provided by Thomson scattering. The *cross-section* will then come almost exclusively from radiation pressure on the electrons, but the *mass* lies almost exclusively in the protons.

There will still be electrostatic forces between the e^-s and the p^+s; if we exert a radiation force on the cloud that is felt mostly by the electrons, they will drag the protons along with them. Thus the approximation is to set $\kappa = \sigma_\mathrm{T}/m_\mathrm{p}$, and we get the following approximation for the Eddington luminosity:

$$\boxed{L_\mathrm{Edd} = \frac{4\pi G M c m_\mathrm{p}}{\sigma_\mathrm{T}}}$$

Note that this approximation for κ is not valid in all situations, especially in stars of all but the highest masses. E.g. in low mass stars the opacity follows Kramer's Law, $\kappa \propto \rho/T^{3.5}$.

Eddington accretion limit

This becomes interesting when the luminosity of the central object is *derived* from matter falling into it. In accretion onto compact objects, infalling matter travels deep into the gravitational potential well of the central object. If it is possible to turn the GPE of the infalling material into heat, huge luminosities can result.

First, however, let us consider the consequences for the *limiting rate* at which such accretion can occur. Suppose a compact object is accreting mass from its surroundings at a rate $\dot{M}$. Next assume that some fraction of the GPE can be radiated away. If we express this as a fraction ϵ of the rest-mass energy, then the luminosity radiated away becomes

$$L = \epsilon \dot{M} c^2$$

This has a profound implication. If our accreting object radiates at more than the Eddington luminosity, even a glut of “fuel” will be blown away by radiation pressure: we get a natural feedback process with a limiting accretion rate. We derive this by setting the accretion luminosity equal to the Eddington luminosity:

$$\epsilon \dot{M} c^2 = \frac{4\pi G M c m_{\mathrm{p}}}{\sigma_{\mathrm{T}}}$$

From which the limiting *Eddington accretion rate* is:

$$\boxed{\dot{M}_{\mathrm{Edd}} = \frac{4\pi G M m_{\mathrm{p}}}{\epsilon c \sigma_{\mathrm{T}}}}$$

Accreting objects in practice

The rather unsubtle catch with this is that we have made no attempt to estimate ϵ. In principle it could be as low as zero: if we simply drop a brick radially into a black hole, it will disappear over the event horizon taking all its energy with it.

However in a realistic astrophysical situation, accreting matter will have angular momentum, forming an *accretion disc*. We will later calculate the canonical value of ϵ which is used for black hole accretion, and estimate the associated temperature and luminosity of the accreting material just before it disappears over the horizon.

First we will recap on the essential properties of black holes, then consider the conditions in the accretion disc.

Properties of black holes

In GR *any* point mass is described by the Schwarzschild (static) or Kerr (rotating) metrics. We use the term black hole to describe an object sufficiently compact (for its mass) that its *event horizon* has noticeable effects on spacetime and matter around the object.

The formal derivation of the radius of the event horizon is beyond the scope of this course (see the GR parts of the theory option for details). However an intuitive approach does give the correct answer for a stationary black hole; we use Newtonian gravity and take the radius to be that at which the escape velocity equals the speed of light. For a point mass M, the escape velocity at radius r is $\sqrt{2GM/r}$. Setting this speed equal to c gives the radius of the event horizon, also known as the *Schwarzschild radius*:

$$R_{\mathsf{S}} = \frac{2GM}{c^2}$$

Note that this scales only with mass; R_{S} for a one-solar-mass object is 3 km. Formally this radius is called the event horizon because it is the furthest distance that a photon starting inside R_{S}can reach; and once photons or matter from outside R_{S} pass beyond it, they cannot escape.

The event horizon is not a solid surface; matter falling inwards passes straight through it. An *external* observer sees any light emitted by the infalling object becoming infinitely redshifted as the object passes over the horizon.

However, due to spacetime curvature near the horizon it is not possible to have a stable circular orbit near the horizon. The *last stable orbit* is at $r = 3R_\mathrm{S}$. For this part of the course we will take this result as given.

Rotating black holes have a more complicated horizon and the last stable orbit is closer in. This may be very important for accretion onto real black holes.

Accretion discs

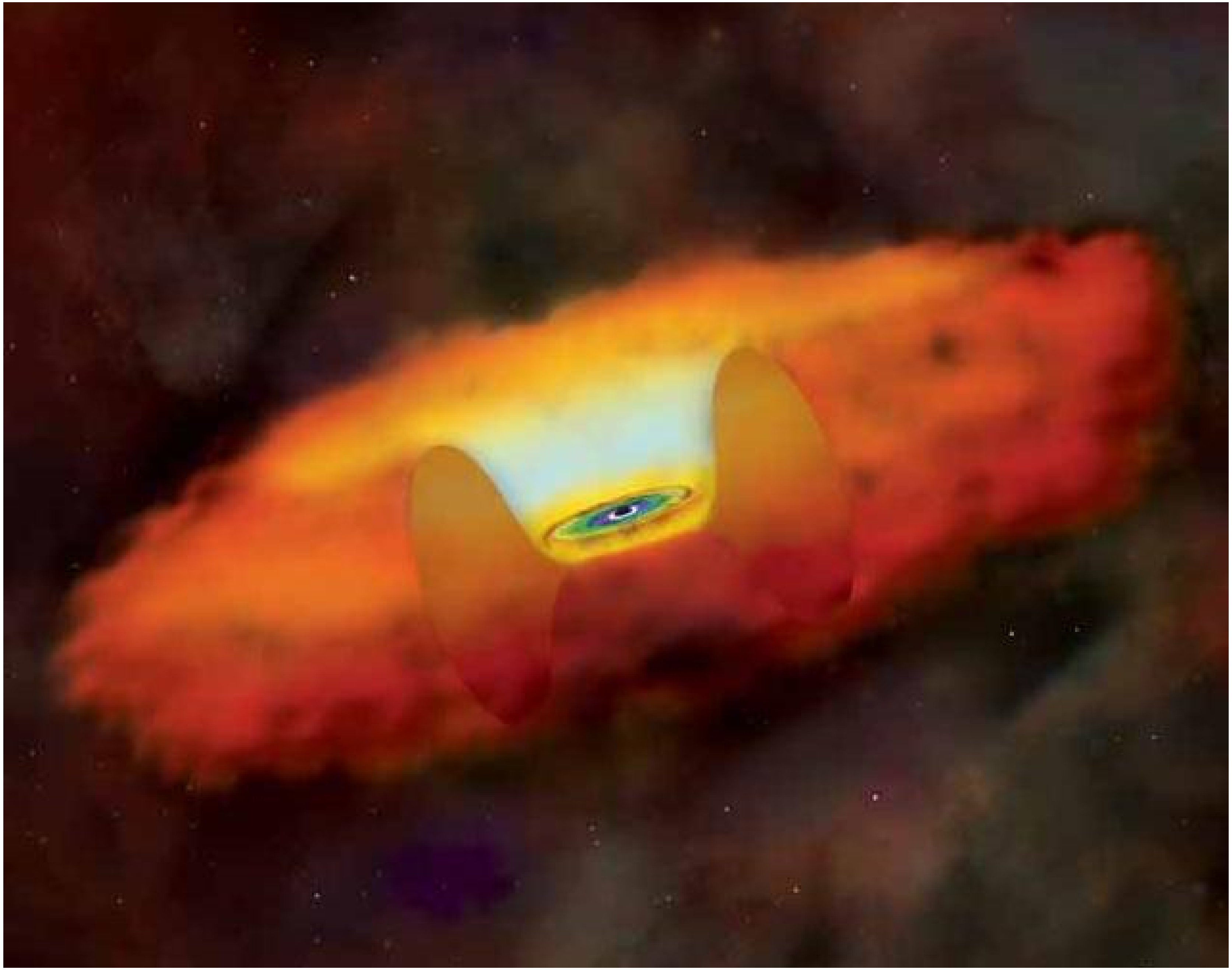
The key to the extraction of energy by material falling into a black hole is to remember that in real astrophysical situation is will have significant *angular momentum*. Gas falling onto a black hole in a binary star system will start with the tangential speed of the binary orbit; gas falling onto the central black hole of an active galaxy may well have an initial tangential velocity of hundreds of km s^{-1} if it begins its descent from the outer regions of the galaxy. A broad-brush picture is:

• An initially large cloud of gas extending well beyond the compact object will, if it has net angular momentum, tend to flatten into a disc. This is because collisions between particles in a direction parallel to the angular momentum **L** vector will tend to sum to zero, whereas collisions perpendicular to the **L** direction will tend to maintain the average circular velocity.

• As the disc becomes sufficiently dense, viscosity *inside* it both *transfers angular momentum outwards* and *heats* the disc. This is how the GPE of the infalling material is radiated away.

• In the most well-studied model, the disc is assumed to be *physically thin* and *optically thick*. This allows the maximum amount of heat to radiate away from the surface of the disk before matter falls into the black hole.

• Eventually we arrive at a reasonably stable state where matter spirals in through the disc, losing angular momentum via friction on its way in, becoming hotter and hotter, until it falls off the inside edge (at the last stable orbit) and crosses over the horizon.



Properties of the thin accretion disc

First of all, the disc must be in hydrostatic equilibrium. In the z direction perpendicular to the plane of the disc we must satisfy

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g_z$$

where g_z is the vertical component of the gravitational acceleration due to the central object,

$$g_z \approx \frac{GM}{R^2}\frac{z}{R}$$

(using small-angle approximation). We can relate P and ρ via the sound speed in the gas, $\mathrm{d}P = c_s^2 \mathrm{d}\rho$, and then integrate to find that *the density in the disc falls off exponentially* with height

$$\rho(z) = \rho_0 \mathrm{exp}\left(-\frac{z^2}{2h^2}\right)$$

with a height scale factor given by $h^2 = c_s^2 R^2/GM$.

Next we consider the speed of rotation. The particles in the disc will have orbits very close to Keplerian, so

$$v_{\mathrm{rot}}^2 = \frac{GM}{R}.$$

The scale height can be re-written in terms of the rotational velocity via

$$h^2 = \frac{c_s^2 R^2}{v_{\mathrm{rot}}^2}$$

so if we have $R \gg h$ we must have $v_{\mathrm{rot}} \gg c_s^2$ and so *the rotation of the disc is highly supersonic*.

Viscosity in the disc

For the accreting material to fall into tighter orbits in the disc there must be an outwards flow of angular momentum—a torque acting on the disc. Take the disc viscosity to be η and consider a radius r in the disc with thickness t and angular velocity ω. The tangential force per unit area exerted by the disc inside r on the disc outside r is given by

$$F = \eta r \frac{\mathrm{d}\omega}{\mathrm{d}r}.$$

This force acts over an area $2\pi r t$ so the total torque between adjacent pieces of the disc is

$$\Gamma = 2\pi r^3 t \eta \frac{\mathrm{d}\omega}{\mathrm{d}r}.$$

Remember torque is rate of change of angular momentum!

Again taking the orbits to be Keplerian we have $\omega = \sqrt{\frac{GM}{r^2}}$, so subbing into the previous equation for the torque we have

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -3\pi\eta t(GMr)^{1/2}$$

which is the rate of change of angular momentum of the inner piece of the disc. This must equal the change of angular momentum duw to inflow of disc material, i.e.

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \dot{m}r^2\omega = \dot{m}(GMr)^{1/2}$$

and so we have a relationship between the accretion rate and the disc viscosity,

$$\dot{m} = 3\pi\eta t$$

The problem with viscosity...*details non-examinable*

The problem arises when we consider the *Reynolds number* of the material in the disc—a measure of how turbulent it is.

$$R \sim \frac{VL}{\nu}$$

Here V and L are characteristic speed and length scales and ν is the kinematic viscosity, η/ρ. We find (see e.g. Longair pp 145–146) that $R \sim 10^{12}$. The flow is *highly* turbulent, and so standard kinetic theory dynamic visosity $\eta = \frac{1}{3}\rho c \lambda$ will make a negligible contribution.

We do not yet understand the precise mechanism of viscosity in accretion discs. Highly turbulent flow helps, but precise calculations are difficult. Magnetic fields will be present and will certainly contribute. Much of the progress to date has come from a neat side-step developed by Shakura and Sunyaev (1972). They invented a parameter

$$\alpha = \frac{\nu}{hc_s}$$

which allows detailed models to be made without knowing the exact mechanism for the viscosity.

Luminosity of a thin accretion disc

Neglecting the energy transport due to viscosity, we can calculate the rate at which accreting material in the disc must lose gravitational potential energy if it is to fall closer to the accreting object. For an annulus between r and $r + \mathrm{d}r$, the energy which must be dissipated will be

$$L(r) = -\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right) = \frac{G\dot{M}M}{2r^2}\mathrm{d}r$$

where $\dot{M}$ is the accretion rate and M is the mass of the central object. Including viscous energy transport we gain a total luminosity three times this value *(non-examinable—see Longair pp 149-150)*.

Temperature structure of a physically thin, optically thick disc

If the disc is optically thick, each annulus between r and $r + \mathrm{d}r$ will radiate as a blackbody with the luminosity derived above. Hence via Stefan's Law (remembering the disc has two surfaces), the annulus at at r will radiate with $2\sigma T^4 \times 2\pi r \mathrm{d}r$. Thus

$$\sigma T^4 = \frac{3GM\dot{M}}{8\pi r^3}$$

and so

$$T(r) = \left(\frac{3G\dot{M}M}{8\pi r^3 \sigma}\right)^{1/4}$$

Spectrum of the thin disc

We are now in a position to describe the form of the overall spectrum of the disc, i.e. the sum of all the black-body contributions at different radii.

$$I_\nu \propto \int_{r_{\text{inner}}}^{r_{\text{outer}}} 2\pi r B_\nu \left\{T(r)\right\} \mathrm{dr}$$

where from lecture 1 we have

$$B_\nu \propto \nu^3 \left(\mathrm{e}^{h\nu/kT} - 1\right)^{-1}$$

From before we have $T \propto r^{-3/4}$ so d$r \propto (1/T)^{1/3}$d$(1/T)$ and we can integrate dT instead of dr. Including B_ν explicitly we therefore have

$$I_\nu \propto \int_{r_{\text{inner}}}^{r_{\text{outer}}} \left(\frac{1}{T}\right)^{4/3} \nu^3 \left(\mathrm{e}^{h\nu/kT} - 1\right)^{-1} \left(\frac{1}{T}\right)^{1/3} \mathrm{d}\left(\frac{1}{T}\right)$$

We can proceed by changing variable $x = (h\nu/kT)$—recall from the second-year thermo problem set where you used the same substitution to derive the functional form $u(T) \propto T^4$. This yields

$$I_\nu \propto \frac{\nu^3}{\nu^{8/3}} \int_{x_{\text{inner}}}^{x_{\text{outer}}} x^{4/3} \left(\mathrm{e}^x - 1\right)^{-1} x^{1/3} \mathrm{d}x$$

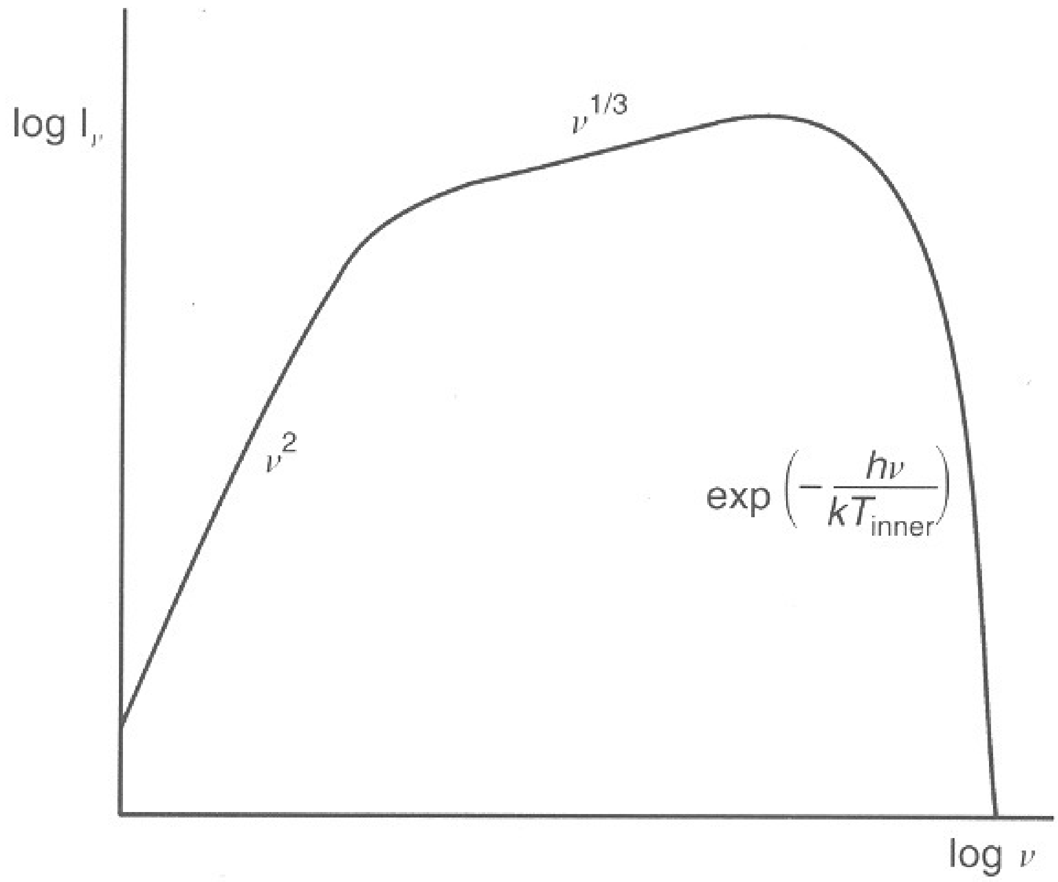
The integral dx is just a numerical constant so we now have the shape of the spectrum over most of its range:

$$I_\nu \propto \nu^{1/3}.$$

Note, though, that the low- and high-frequency ends will have a different form:

- From the outer edge of the disc we will see the Rayleigh-Jeans tail of T_{outer}, $I_\nu \propto \nu^2$.
- From the inner edge, an exponential cut-off $I_\nu \propto \left(\mathrm{e}^{-h\nu/kT_{\text{inner}}}\right)$.

Theoretical spectrum of thin accretion disc.



Total luminosity of the thin disc

Let's now estimate ϵ. If we approximate with a Newtonian potential, take the particle to have started its trip at $r = \infty$ and total energy zero, and calculate the total energy it has on the last stable orbit. The amount of GPE which must be lost (by radiation) is equivalent to 1/12 of the rest-mass energy of the particle.

For the best possible case—the closest orbit around a rapidly-rotating black hole—the efficiency rises to a whopping 0.42. Compare this to nuclear fusion in stars, which has an efficiency of only 0.7 percent!

Hence in practice, astronomers usually adopt an approximate value of $\epsilon = 0.1$ for accretion onto black holes.

Example: estimating a quasar accretion rate

Suppose we observe a quasar to have a total power output of 10^{40} W. We are now in a position to estimate the mass of the central black hole and the rate at which its mass is increasing.

First let us assume that the accretion is Eddington limited. From our equation for the Eddington luminosity we have

$$L_{\mathrm{Edd}} = \frac{4\pi G M c m_{\mathrm{p}}}{\sigma_{\mathrm{T}}}$$

from which

$$M = 7 \times 10^8 M_\odot.$$

And from our Eddington accretion rate, using $\epsilon = 0.1$ we have

$$\dot{M}_{\mathrm{Edd}} = \frac{4\pi G M m_{\mathrm{p}}}{\epsilon c \sigma_{\mathrm{T}}} \approx 3 M_\odot \mathrm{yr}^{-1}$$

Getting round the Eddinton limit

The accretion may not always be Eddington-limited. It is, for example, possible to achieve $\dot{M}$ much greater than would be inferred by using the Eddington luminosity with ϵ = 0.1, by making the disc physically thick, and very low density, so that it is optically thin and matter doesn't have time to radiate away so much energy before it falls over the horizon. This has the advantages of allowing black holes to grow at a very high rate in the early Universe (recall the B3 problem set from last year), and also of providing "funnels" which could be a mechanism for collimating outflows from accreting objects.

Unfortunately simple analytical models of these discs are unstable, but the advantages of thick discs are so great that much effort is put into modelling them numerically... including the effects of strong magnetic fields.

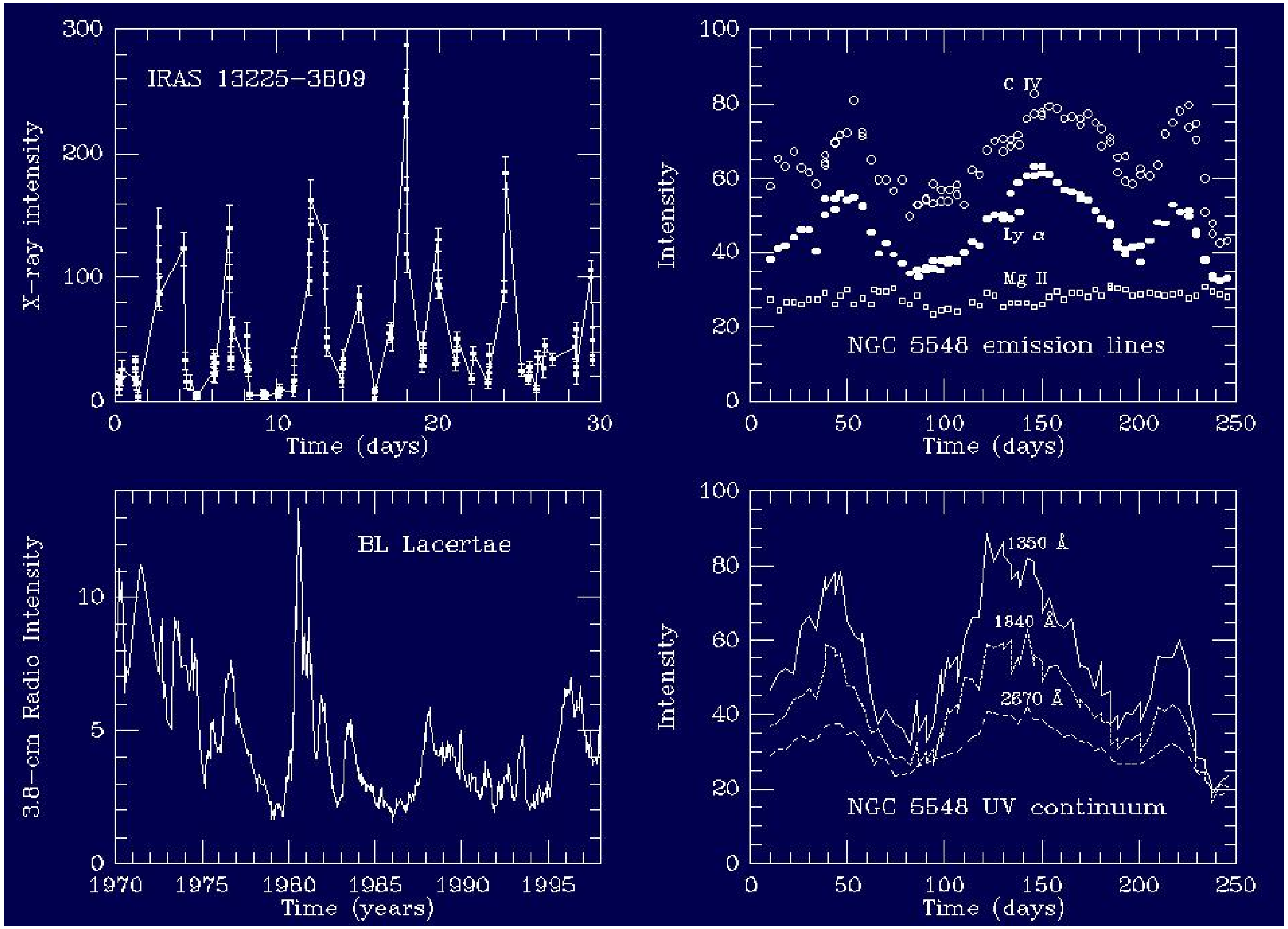
It is also possible for an object to have a luminosity significantly greater than the Eddington luminosity:

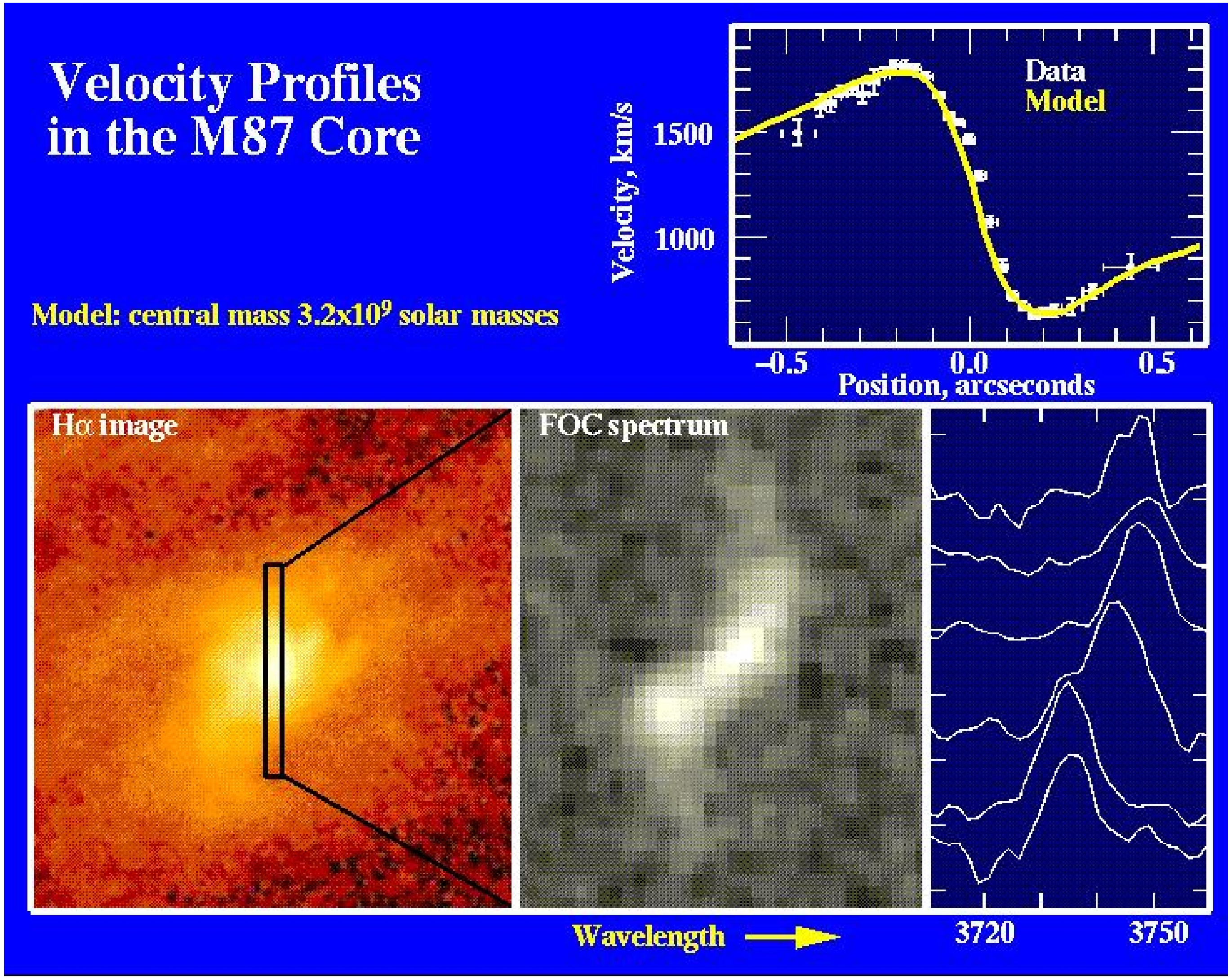
- In supernovae (somewhat trivially!)
- Where spherical symmetry is broken, with extremely collimated radiation in a direction different to the accretion direction
- Where accretion is not steady, e.g. bursts or radiation emitted when discrete clouds of matter fall onto a neutron star or white dwarf.

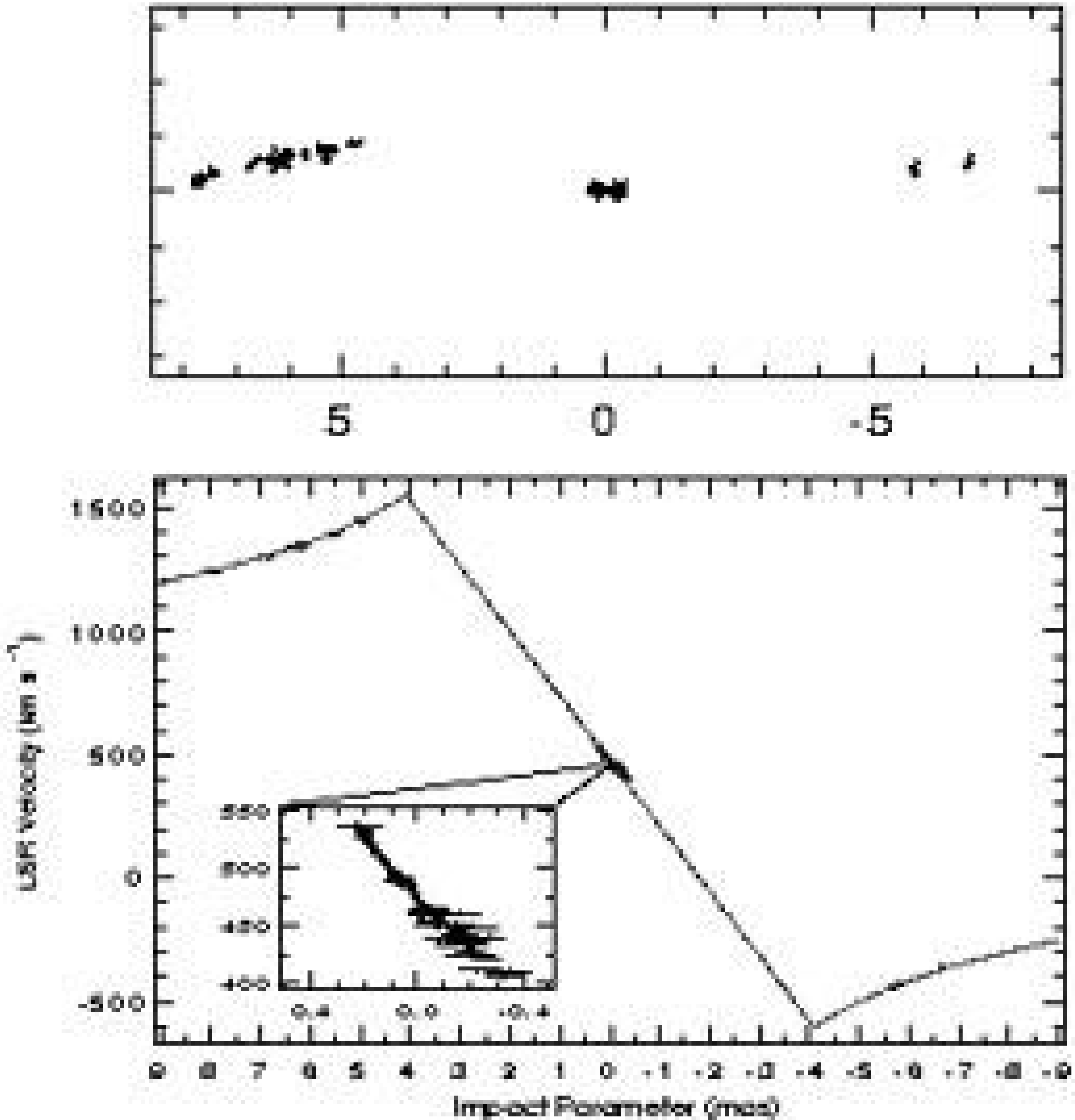
Evidence for black holes in AGN

There are several canonical pieces of evidence that supermassive black holes really are there at the heart of AGN. Among these are:

- Variability (in combination with Eddington luminosity).
- Stellar velocity dispersions.
- Rotation speeds inferred from emission lines.
- (Perhaps most spectacularly) X-ray line profiles.


IRAS 13225-3809
X-ray intensity
Time (days)
NGC 5548 emission lines
C IV
Ly α
Mg II
Intensity
BL Lacertae
3.8-cm Radio Intensity
Time (years)
NGC 5548 UV continuum
1350 Å
1840 Å
2670 Å


Velocity Profiles
in the M87 Core
Model: central mass 3.2x10^9 solar masses
Data
Model
Velocity, km/s
1500
1000
–0.5
0.0
0.5
Position, arcseconds
Hα image
FOC spectrum
Wavelength
3720
3750


5
0
-5
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LSR Velocity (km s^-1)
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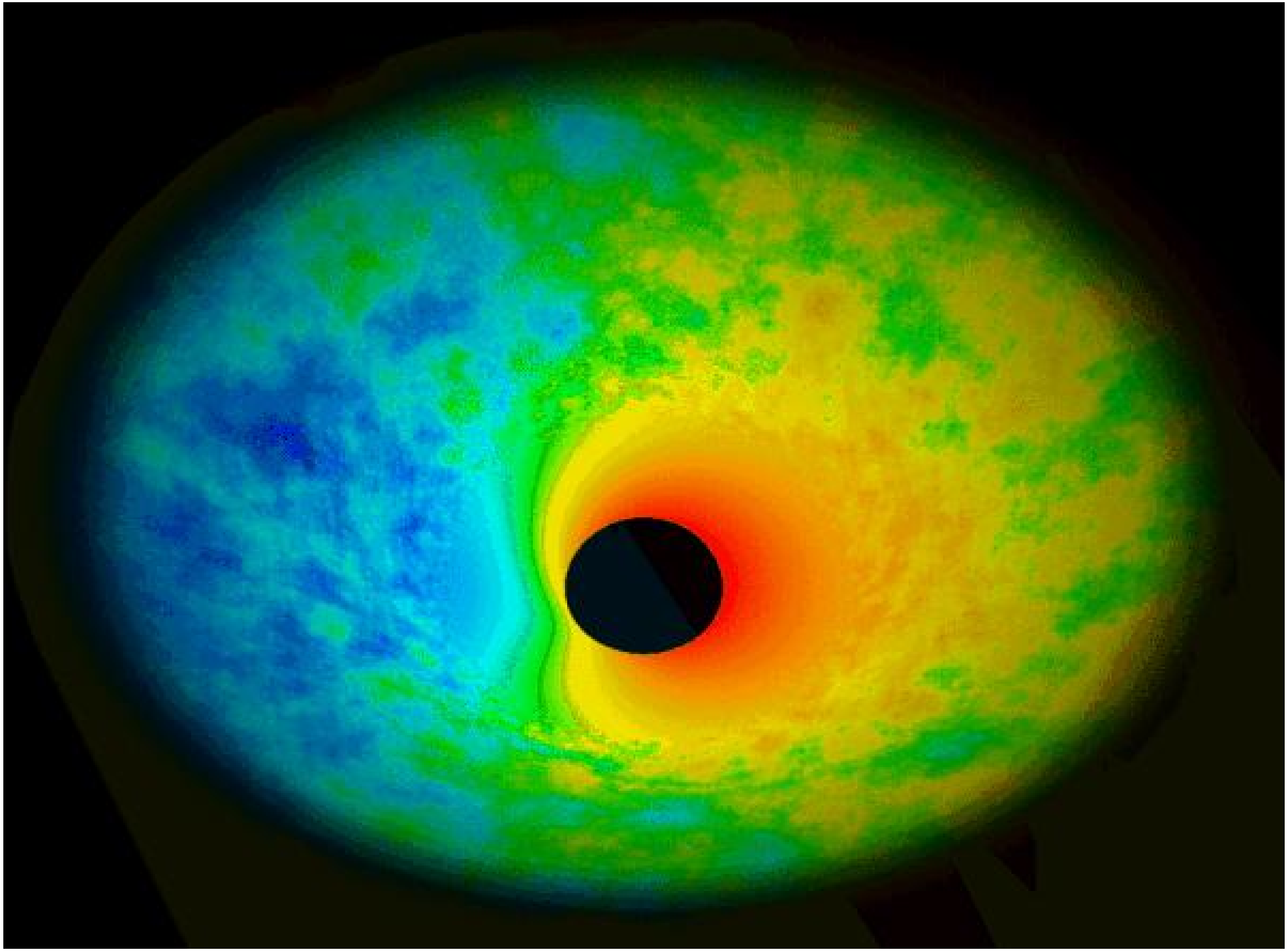
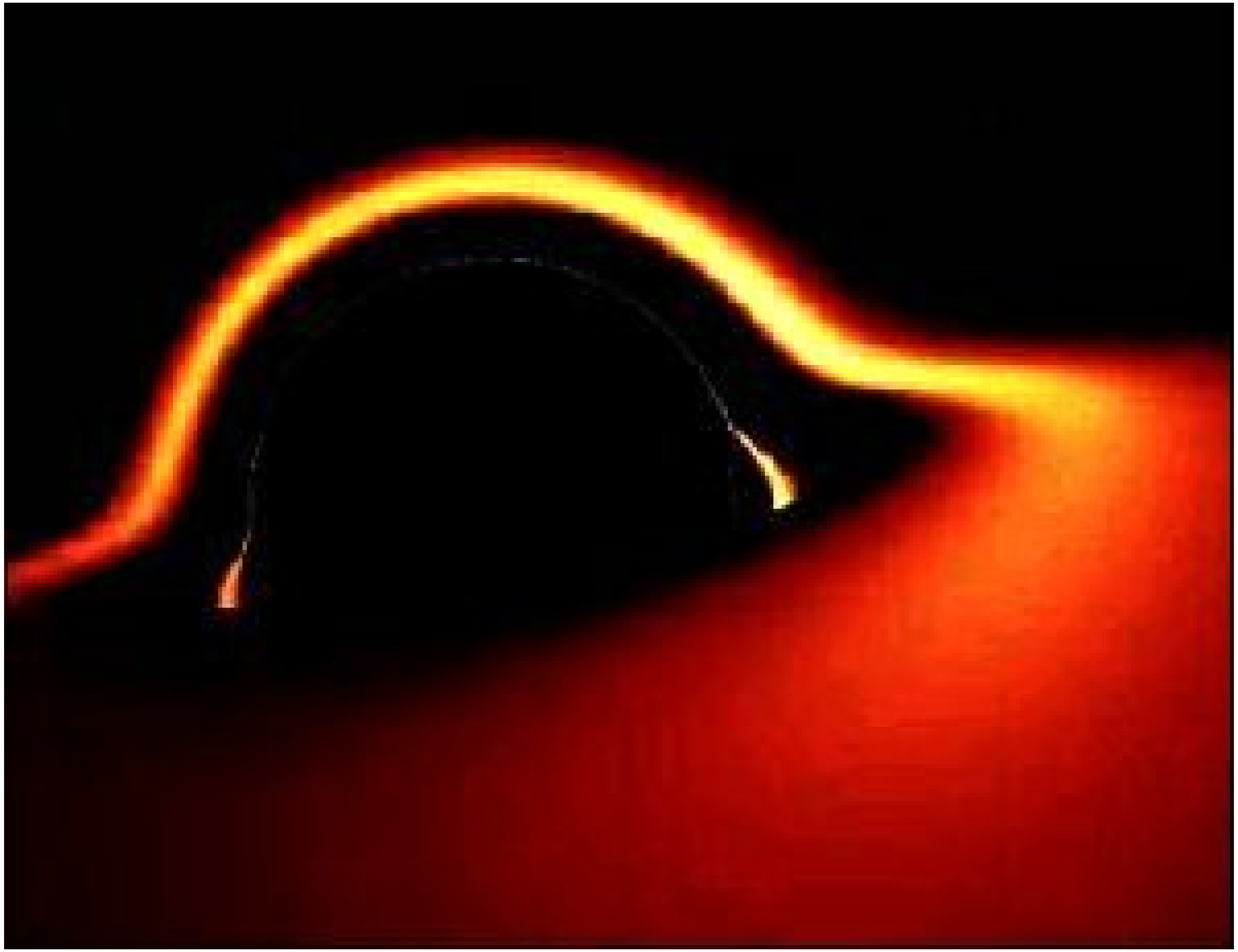
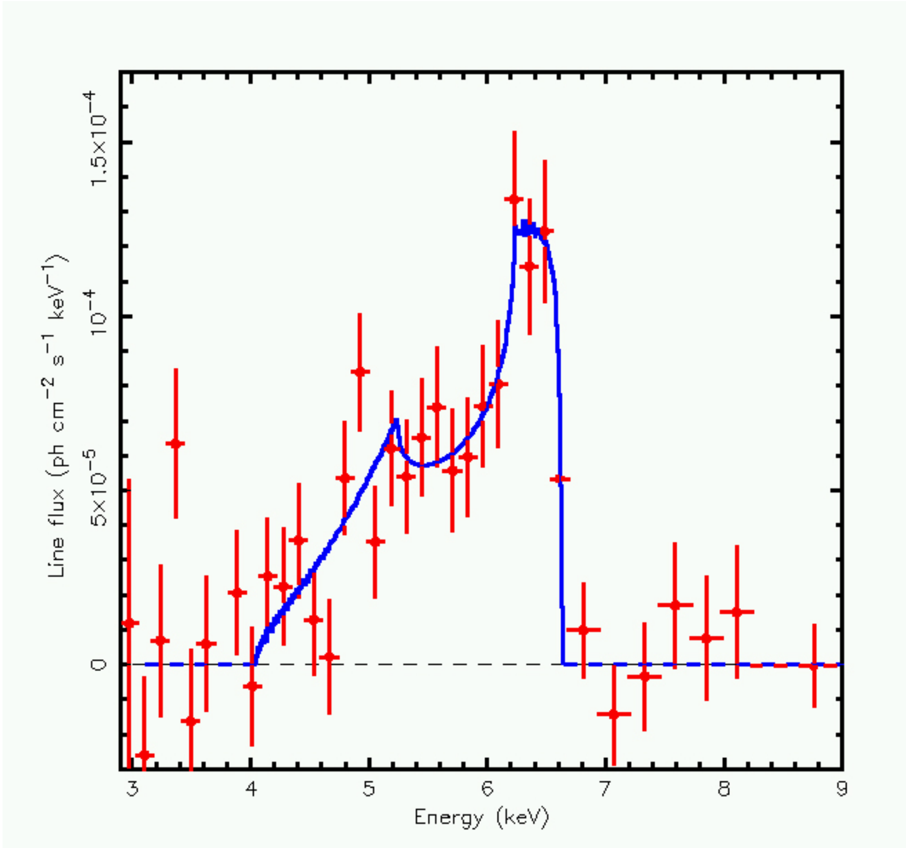
Impact Parameter (mas)







Line flux (ph cm^{-2} s^{-1} keV^{-1})
1.5×10^{-4}
10^{-4}
5×10^{-5}
0
3
4
5
6
7
8
9
Energy (keV)